Math 221: Final Exam Extras and Theory Review Sheet

Ratio/Proportion Problems: Solve the following word problems. For problems 2-3, solve the proportions by using a table or figure instead of writing the proportion and solving. For problems 4-5, solve by using a proportion.

1. There are 27 girls in a class and 36 boys in a class. What is the ratio of girls to boys? Girls to class? Boys to class? Simplify your ratios.
2. The ratio of gravy to mashed potatoes on your plate is 1:8. If you got 2 cups of mashed potatoes, how many cups of gravy did you put on your mashed potatoes?
3. The ratio of water to uncooked rice in order to steam the rice properly is 2:1. What is the ratio of the total product to the amount of uncooked rice? How much water do you need if you want to steam 4 cups of uncooked rice? How much total product do you get?
4. The ratio of pollen particles to total particles making up the air reached 1,300:1,000,000 last year. How many pollen particles were among 1,800,000 total particles last year?
5. The ratio of teenagers to adults at a banquet is 3:2. If there are 180 teenagers at the banquet, how many people total are at the banquet?

Converting Fractions to Percents: Convert the following fractions to percents by writing them over 100 if possible. If it is not possible, use long division to convert them to a decimal first.

1. $\frac{21}{25}$
2. $\frac{21}{30}$
3. $\frac{17}{20}$

Converting Percents to Fractions: Convert the following percents to fractions.

1. 64%
2. 15.5%
3. $16.\overline{6}\%$

Percent Problems: Solve the following word problems.

1. Marc had 188 boxes of candy to sell, but he only managed to sell 25% of them. How many did he sell?
2. After income taxes of 20%, Michael made $720. How much was his original paycheck worth?
3. If a dress that normally sells for $45 is on sale for $27, what is the percent discount?
4. Mort bought a house for $165,000. If the house is now worth $198,000, how what was the percent increase of the price of his house?
5. Gail received a 5% raise at the beginning of the year. If she now makes $19929, how much was she making last year?

Test 2 Material Explanations: Use an example to explain problems 1-4. On the actual exam, you will be given an example and asked a specific question as to why something works.

1. Explain why the standard algorithms of addition, subtraction, and multiplication work using base 10 blocks.
2. Explain why the equal additions algorithm works.
3. Explain why the lattice algorithms of addition and multiplication hold the correct place values together.
4. Explain why long division works using the partition method.
5. Explain why the divisibility tests for 2, 4, 5, 6, 8, and 10 make sense for any number.

Test 3 Material Explanations: Use an example to explain. On the actual exam, you will be given an example and asked a specific question as to why something works.

1. Explain why the definition of multiplication of fractions makes sense using a figure.
2. Explain why zeros at the end of a decimal don't matter.
3. Explain why we choose our decimal place where we do in multiplication of decimals.
4. Explain why we add a decimal point and a zero when we divide an integer by an integer and don't have a zero remainder.
5. Explain why we add additional zeros when we are dividing a decimal by an integer and don't have a zero remainder.
6. Explain why we can move the decimal place of the dividend and divisor an equal number of times in order to divide a decimal by a decimal.

Proofs: Prove all of the following theorems.

1. Two fractions $\frac{a}{b}$ and $\frac{c}{d}$ are equivalent if and only if $ad = bc$.
2. If a, b, c, and d are integers with $b, d > 0$, then $\frac{a}{b} > \frac{c}{d}$ if and only if $ad > bc$.
3. If a, b, c, and d are integers with $b, d > 0$, then $\frac{a}{b} < \frac{c}{d}$ if and only if $ad < bc$.
4. If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$
5. If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$
6. If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers and $\frac{c}{d} \neq 0$, then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$.
7. A fraction in simplest form can be written as a terminating decimal if and only if the only primes that appear in the factorization of its denominator are 2 and 5.

Sets: Define the following sets.

1. $\mathbb{N}$
2. $\mathbb{W}$
3. $\mathbb{Z}$
4. $\mathbb{Q}$
5. $\mathbb{R}$

Properties: Define the following properties. For 1-5, define them for addition, subtraction, multiplication, and division.

1. Closure
2. Commutative
3. Associative
4. Identity
5. Inverse
6. Distributive
7. Zero Product

8. Using $\mathbb{N}$, $\mathbb{W}$, $\mathbb{Z}$, $\mathbb{Q}$, and $\mathbb{R}$, state which of the above properties hold with that set and (1) all 4 operations for 1-5, (2) the normal operations for 6-7. These will be true/false questions on the final.

Example Property Questions: Determine if the following statements are true or false. If it is false, give a specific example that shows why.

1. $\mathbb{Z}$ has the commutative property with addition.
2. $\mathbb{R}$ has the inverse property with multiplication.
3. $\mathbb{W}$ has the closure property with subtraction.
4. $\mathbb{N}$ has the zero product property.
5. $\mathbb{Q}$ has the distributive property.
6. $\mathbb{R}$ has the identity property with division.
7. $\mathbb{W}$ has the associative property with addition.
8. $\mathbb{Z}$ has the inverse property with multiplication.
9. $\mathbb{Q}$ has the identity property with subtraction.
10. $\mathbb{N}$ has the closure property with multiplication.
11. $\mathbb{R}$ has the zero product property.
12. $\mathbb{Q}$ has the commutative property with subtraction.

Questions from the Classroom: Answer the following questions.

1. Sue claims the following is true by the distributive property: Where a and b are whole numbers, $3\,(ab) = (3a)\,(3b)$. How might you help her?
2. A student claims that to divide a number with the units digit 0 by 10, she just crosses out the 0 to get the answer. She wants to know if this is always true and why and if the 0 has to be the units digit. How do you respond?
3. Explain why $b - a$ and $a - b$ are additive inverses of each other.
4. An eighth-grade student claims she can prove that subtraction of integers is commutative. She points out that if a and b are integers, then $a - b = a + {}^{-}b$. Since addition is commutative, so is subtraction. What is your response?
5. A student computes ${}^{-}8 - 2\,({}^{-}3)$ by writing ${}^{-}10\,({}^{-}3) = 30$. How would you help this student?
6. Bob says that to check if a number is prime he just uses the divisibility rules he knows for 2, 3, 4, 5, 6, 7, 8, 9, and 10. He says that if the number is not divisible by these numbers, then it is prime. How do you respond?
7. Iris claims that if we have two positive rational numbers, the one with the greater numerator is greater. How do you respond?
8. To show $2\frac{3}{4} = \frac{11}{4}$, the teacher drew the following picture. Ken said this show a picture of $\frac{11}{12}$, not $\frac{11}{4}$. What is Ken thinking and how should the teacher respond?

9. A student claims that division always makes things smaller so $5 \div \frac{1}{2}$ can't be 10 because that is greater than the number she started with. How would you help this student?
10. A student claims that 0.36 is greater than 0.9 because 36 is greater than 9. How do you respond?
11. A student claims that $\frac{9443}{9444}$ and $\frac{9444}{9445}$ are equal because both display 0.9998941 on his scientific calculator when the divisions are performed. How do you respond?
12. A student claims that since it is possible to have a 150% increase in price, it is possible to have a 150% decrease in price. How do you respond?
13. A student asks if 90% means 90 out of 100, how can she possibly score a 90% on a test that only has 20 questions? How do you answer her?